

## C4 VECTORS

## Answers - Worksheet D

1    **a** =  $3 + 2 = 5$                       **b** =  $12 - 5 = 7$                       **c** =  $-5 + 4 = -1$

2     $(\mathbf{i} + 4\mathbf{j}) \cdot (8\mathbf{i} - 2\mathbf{j}) = 8 - 8 = 0$   
 $\therefore$  perpendicular

3    **a**     $\begin{pmatrix} 3 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} c \\ 3 \end{pmatrix} = 0$                       **b**     $\begin{pmatrix} 2 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ c \end{pmatrix} = 0$                       **c**     $\begin{pmatrix} 2 \\ -5 \end{pmatrix} \cdot \begin{pmatrix} c \\ -4 \end{pmatrix} = 0$   
 $3c - 3 = 0$                        $6 + c = 0$                        $2c + 20 = 0$   
 $c = 1$                        $c = -6$                        $c = -10$

4    **a**     $|4\mathbf{i} - 3\mathbf{j}| = \sqrt{16+9} = 5$ ,  $|8\mathbf{i} + 6\mathbf{j}| = \sqrt{64+36} = 10$   
 $(4\mathbf{i} - 3\mathbf{j}) \cdot (8\mathbf{i} + 6\mathbf{j}) = 32 - 18 = 14$   
 $\therefore$  angle =  $\cos^{-1} \frac{14}{5 \times 10} = \cos^{-1} \frac{7}{25} = 73.7^\circ$

**b**     $|7\mathbf{i} + \mathbf{j}| = \sqrt{49+1} = 5\sqrt{2}$ ,  $|2\mathbf{i} + 6\mathbf{j}| = \sqrt{4+36} = 2\sqrt{10}$   
 $(7\mathbf{i} + \mathbf{j}) \cdot (2\mathbf{i} + 6\mathbf{j}) = 14 + 6 = 20$   
 $\therefore$  angle =  $\cos^{-1} \frac{20}{5\sqrt{2} \times 2\sqrt{10}} = \cos^{-1} \frac{1}{\sqrt{5}} = 63.4^\circ$

**c**     $|4\mathbf{i} + 2\mathbf{j}| = \sqrt{16+4} = 2\sqrt{5}$ ,  $|-5\mathbf{i} + 2\mathbf{j}| = \sqrt{25+4} = \sqrt{29}$   
 $(4\mathbf{i} + 2\mathbf{j}) \cdot (-5\mathbf{i} + 2\mathbf{j}) = -20 + 4 = -16$   
 $\therefore$  angle =  $\cos^{-1} \frac{-16}{2\sqrt{5} \times \sqrt{29}} = \cos^{-1} \left(-\frac{8}{\sqrt{5}\sqrt{29}}\right) = 131.6^\circ$

5     $\overrightarrow{BA} = (9\mathbf{i} + \mathbf{j}) - (3\mathbf{i} - \mathbf{j}) = 6\mathbf{i} + 2\mathbf{j}$   
 $\overrightarrow{BC} = (5\mathbf{i} - 2\mathbf{j}) - (3\mathbf{i} - \mathbf{j}) = 2\mathbf{i} - \mathbf{j}$   
 $|\overrightarrow{BA}| = \sqrt{36+4} = 2\sqrt{10}$ ,  $|\overrightarrow{BC}| = \sqrt{4+1} = \sqrt{5}$   
 $\overrightarrow{BA} \cdot \overrightarrow{BC} = (6\mathbf{i} + 2\mathbf{j}) \cdot (2\mathbf{i} - \mathbf{j}) = 12 - 2 = 10$   
 $\therefore \angle ABC = \cos^{-1} \frac{10}{2\sqrt{10} \times \sqrt{5}} = \cos^{-1} \frac{1}{\sqrt{2}} = 45^\circ$

6    **a** =  $3 + 2 + 8 = 13$                       **b** =  $6 + 6 - 2 = 10$   
**c** =  $-5 + 0 - 6 = -11$                       **d** =  $-3 + 22 + 32 = 51$   
**e** =  $27 - 28 - 1 = -2$                       **f** =  $0 + 9 + 0 = 9$

7    **a** =  $(2\mathbf{i} + \mathbf{j} - 3\mathbf{k}) \cdot (\mathbf{i} + 5\mathbf{j} - \mathbf{k}) = 2 + 5 + 3 = 10$   
**b** =  $(2\mathbf{i} + \mathbf{j} - 3\mathbf{k}) \cdot (6\mathbf{i} - 2\mathbf{j} - 3\mathbf{k}) = 12 - 2 + 9 = 19$   
**c**     $\mathbf{q} + \mathbf{r} = (\mathbf{i} + 5\mathbf{j} - \mathbf{k}) + (6\mathbf{i} - 2\mathbf{j} - 3\mathbf{k}) = 7\mathbf{i} + 3\mathbf{j} - 4\mathbf{k}$   
 $\mathbf{p} \cdot (\mathbf{q} + \mathbf{r}) = (2\mathbf{i} + \mathbf{j} - 3\mathbf{k}) \cdot (7\mathbf{i} + 3\mathbf{j} - 4\mathbf{k}) = 14 + 3 + 12 = 29$   
 $\mathbf{p} \cdot \mathbf{q} + \mathbf{p} \cdot \mathbf{r} = 10 + 19 = 29$   
 $\therefore \mathbf{p} \cdot (\mathbf{q} + \mathbf{r}) = \mathbf{p} \cdot \mathbf{q} + \mathbf{p} \cdot \mathbf{r}$

8    **a** =  $\mathbf{p} \cdot \mathbf{q} + \mathbf{p} \cdot \mathbf{r} + \mathbf{p} \cdot \mathbf{q} - \mathbf{p} \cdot \mathbf{r}$                       **b** =  $\mathbf{p} \cdot \mathbf{q} + \mathbf{p} \cdot \mathbf{r} + \mathbf{q} \cdot \mathbf{r} - \mathbf{q} \cdot \mathbf{p}$   
=  $2\mathbf{p} \cdot \mathbf{q}$                       =  $\mathbf{p} \cdot \mathbf{q} + \mathbf{p} \cdot \mathbf{r} + \mathbf{q} \cdot \mathbf{r} - \mathbf{p} \cdot \mathbf{q}$   
   =  $\mathbf{p} \cdot \mathbf{r} + \mathbf{q} \cdot \mathbf{r}$   
   =  $(\mathbf{p} + \mathbf{q}) \cdot \mathbf{r}$

9  $(5\mathbf{i} - 3\mathbf{j} + 2\mathbf{k}) \cdot (3\mathbf{i} + \mathbf{j} - 6\mathbf{k}) = 15 - 3 - 12 = 0$   
 $\therefore$  perpendicular

10  $\overrightarrow{BA} = (3\mathbf{i} + 4\mathbf{j} - 6\mathbf{k}) - (\mathbf{i} + 5\mathbf{j} - 2\mathbf{k}) = 2\mathbf{i} - \mathbf{j} - 4\mathbf{k}$   
 $\overrightarrow{BC} = (8\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}) - (\mathbf{i} + 5\mathbf{j} - 2\mathbf{k}) = 7\mathbf{i} - 2\mathbf{j} + 4\mathbf{k}$   
 $\overrightarrow{BA} \cdot \overrightarrow{BC} = (2\mathbf{i} - \mathbf{j} - 4\mathbf{k}) \cdot (7\mathbf{i} - 2\mathbf{j} + 4\mathbf{k}) = 14 + 2 - 16 = 0$   
 $\therefore \overrightarrow{BA}$  and  $\overrightarrow{BC}$  are perpendicular  $\therefore \angle ABC = 90^\circ$

11 a  $(2\mathbf{i} + 3\mathbf{j} + \mathbf{k}) \cdot (c\mathbf{i} - 3\mathbf{j} + \mathbf{k}) = 0$   
 $2c - 9 + 1 = 0$   
 $c = 4$

b  $(-5\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}) \cdot (c\mathbf{i} - \mathbf{j} + 3c\mathbf{k}) = 0$   
 $-5c - 3 + 6c = 0$   
 $c = 3$

c  $(c\mathbf{i} - 2\mathbf{j} + 8\mathbf{k}) \cdot (c\mathbf{i} + c\mathbf{j} - 3\mathbf{k}) = 0$   
 $c^2 - 2c - 24 = 0$   
 $(c + 4)(c - 6) = 0$   
 $c = -4, 6$

d  $(3c\mathbf{i} + 2\mathbf{j} + c\mathbf{k}) \cdot (5\mathbf{i} - 4\mathbf{j} + 2c\mathbf{k}) = 0$   
 $15c - 8 + 2c^2 = 0$   
 $2c^2 + 15c - 8 = 0$   
 $(2c - 1)(c + 8) = 0$   
 $c = -8, \frac{1}{2}$

12 a  $\left| \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix} \right| = \sqrt{1+4+4} = 3$ ,  $\left| \begin{pmatrix} 8 \\ 1 \\ -4 \end{pmatrix} \right| = \sqrt{64+1+16} = 9$ ,  $\begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix} \cdot \begin{pmatrix} 8 \\ 1 \\ -4 \end{pmatrix} = 8 + 2 + 8 = 18$   
 $\therefore \cos \theta = \frac{18}{3 \times 9} = \frac{2}{3}$

b  $\left| \begin{pmatrix} 4 \\ 1 \\ -2 \end{pmatrix} \right| = \sqrt{16+1+4} = \sqrt{21}$ ,  $\left| \begin{pmatrix} -2 \\ 3 \\ -6 \end{pmatrix} \right| = \sqrt{4+9+36} = 7$ ,  $\begin{pmatrix} 4 \\ 1 \\ -2 \end{pmatrix} \cdot \begin{pmatrix} -2 \\ 3 \\ -6 \end{pmatrix} = -8 + 3 + 12 = 7$   
 $\therefore \cos \theta = \frac{7}{\sqrt{21} \times 7} = \frac{1}{\sqrt{21}}$  or  $\frac{1}{21}\sqrt{21}$

c  $\left| \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} \right| = \sqrt{1+4+1} = \sqrt{6}$ ,  $\left| \begin{pmatrix} 1 \\ -7 \\ 2 \end{pmatrix} \right| = \sqrt{1+49+4} = 3\sqrt{6}$ ,  $\begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -7 \\ 2 \end{pmatrix} = 1 - 14 - 2 = -15$   
 $\therefore \cos \theta = \frac{-15}{\sqrt{6} \times 3\sqrt{6}} = -\frac{5}{6}$

d  $\left| \begin{pmatrix} 5 \\ -3 \\ 4 \end{pmatrix} \right| = \sqrt{25+9+16} = 5\sqrt{2}$ ,  $\left| \begin{pmatrix} 3 \\ -4 \\ -1 \end{pmatrix} \right| = \sqrt{9+16+1} = \sqrt{26}$ ,  $\begin{pmatrix} 5 \\ -3 \\ 4 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ -4 \\ -1 \end{pmatrix} = 15 + 12 - 4 = 23$   
 $\therefore \cos \theta = \frac{23}{5\sqrt{2} \times \sqrt{26}} = \frac{23}{10\sqrt{13}}$  or  $\frac{23}{130}\sqrt{13}$

13 a  $|(3\mathbf{i} - 4\mathbf{k})| = \sqrt{9+16} = 5$   
 $|(7\mathbf{i} - 4\mathbf{j} + 4\mathbf{k})| = \sqrt{49+16+16} = 9$   
 $(3\mathbf{i} - 4\mathbf{k}) \cdot (7\mathbf{i} - 4\mathbf{j} + 4\mathbf{k}) = 21 + 0 - 16 = 5$   
 $\therefore \text{angle} = \cos^{-1} \frac{5}{5 \times 9} = \cos^{-1} \frac{1}{9} = 83.6^\circ$

b  $|(2\mathbf{i} - 6\mathbf{j} + 3\mathbf{k})| = \sqrt{4+36+9} = 7$   
 $|(i - 3j - k)| = \sqrt{1+9+1} = \sqrt{11}$   
 $(2\mathbf{i} - 6\mathbf{j} + 3\mathbf{k}) \cdot (\mathbf{i} - 3\mathbf{j} - \mathbf{k}) = 2 + 18 - 3 = 17$   
 $\therefore \text{angle} = \cos^{-1} \frac{17}{7 \times \sqrt{11}} = 42.9^\circ$

c  $|(6\mathbf{i} - 2\mathbf{j} - 9\mathbf{k})| = \sqrt{36+4+81} = 11$   
 $|(3\mathbf{i} + \mathbf{j} + 4\mathbf{k})| = \sqrt{9+1+16} = \sqrt{26}$   
 $(6\mathbf{i} - 2\mathbf{j} - 9\mathbf{k}) \cdot (3\mathbf{i} + \mathbf{j} + 4\mathbf{k}) = 18 - 2 - 36 = -20$   
 $\therefore \text{angle} = \cos^{-1} \frac{-20}{11 \times \sqrt{26}} = 110.9^\circ$

d  $|(i + 5j - 3k)| = \sqrt{1+25+9} = \sqrt{35}$   
 $|(-3i - 4j + 2k)| = \sqrt{9+16+4} = \sqrt{29}$   
 $(i + 5j - 3k) \cdot (-3i - 4j + 2k) = -3 - 20 - 6 = -29$   
 $\therefore \text{angle} = \cos^{-1} \frac{-29}{\sqrt{35} \times \sqrt{29}} = \cos^{-1} \left( -\sqrt{\frac{29}{35}} \right) = 155.5^\circ$

$$14 \quad \mathbf{a} \quad \overrightarrow{BA} = (7+1)\mathbf{i} + (2-6)\mathbf{j} + (-2+3)\mathbf{k} = 8\mathbf{i} - 4\mathbf{j} + \mathbf{k}$$

$$\overrightarrow{BC} = (-3+1)\mathbf{i} + (1-6)\mathbf{j} + (2+3)\mathbf{k} = -2\mathbf{i} - 5\mathbf{j} + 5\mathbf{k}$$

$$\mathbf{b} \quad |\overrightarrow{BA}| = \sqrt{64+16+1} = 9$$

$$|\overrightarrow{BC}| = \sqrt{4+25+25} = 3\sqrt{6}$$

$$\overrightarrow{BA} \cdot \overrightarrow{BC} = (8\mathbf{i} - 4\mathbf{j} + \mathbf{k}) \cdot (-2\mathbf{i} - 5\mathbf{j} + 5\mathbf{k}) = -16 + 20 + 5 = 9$$

$$\therefore \angle ABC = \cos^{-1} \frac{9}{9 \times 3\sqrt{6}} = \cos^{-1} \frac{1}{3\sqrt{6}} = 82.2^\circ$$

$$\mathbf{c} = \frac{1}{2} \times 9 \times 3\sqrt{6} \times \sin 82.18^\circ = 32.8$$

$$15 \quad \mathbf{a} \quad \overrightarrow{AB} = (4\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}) - (3\mathbf{i} - 2\mathbf{j} - \mathbf{k}) = \mathbf{i} + 5\mathbf{j} - \mathbf{k}$$

$$\overrightarrow{AC} = (2\mathbf{i} - \mathbf{j}) - (3\mathbf{i} - 2\mathbf{j} - \mathbf{k}) = -\mathbf{i} + \mathbf{j} + \mathbf{k}$$

$$|\overrightarrow{AB}| = \sqrt{1+25+1} = 3\sqrt{3}$$

$$|\overrightarrow{AC}| = \sqrt{1+1+1} = \sqrt{3}$$

$$\overrightarrow{AB} \cdot \overrightarrow{AC} = (\mathbf{i} + 5\mathbf{j} - \mathbf{k}) \cdot (-\mathbf{i} + \mathbf{j} + \mathbf{k}) = -1 + 5 - 1 = 3$$

$$\cos(\angle BAC) = \frac{3}{3\sqrt{3} \times \sqrt{3}} = \frac{1}{3}$$

$$\mathbf{b} \quad \sin^2(\angle BAC) = 1 - \left(\frac{1}{3}\right)^2 = \frac{8}{9}$$

$$\sin(\angle BAC) = \sqrt{\frac{8}{9}} = \frac{2}{3}\sqrt{2}$$

$$\text{area} = \frac{1}{2} \times 3\sqrt{3} \times \sqrt{3} \times \frac{2}{3}\sqrt{2} = 3\sqrt{2}$$

$$16 \quad \mathbf{a} \quad \left| \begin{pmatrix} 4 \\ -4 \\ 2 \end{pmatrix} \right| = \sqrt{16+16+4} = 6, \quad \left| \begin{pmatrix} 8 \\ 0 \\ -6 \end{pmatrix} \right| = \sqrt{64+36} = 10, \quad \begin{pmatrix} 4 \\ -4 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} 8 \\ 0 \\ -6 \end{pmatrix} = 32 + 0 - 12 = 20$$

$$\therefore \text{acute angle} = \cos^{-1} \left| \frac{20}{6 \times 10} \right| = \cos^{-1} \frac{1}{3} = 70.5^\circ$$

$$\mathbf{b} \quad \left| \begin{pmatrix} 6 \\ -1 \\ -18 \end{pmatrix} \right| = \sqrt{36+1+324} = 19, \quad \left| \begin{pmatrix} 4 \\ -12 \\ 3 \end{pmatrix} \right| = \sqrt{16+144+9} = 13, \quad \begin{pmatrix} 6 \\ -1 \\ -18 \end{pmatrix} \cdot \begin{pmatrix} 4 \\ -12 \\ 3 \end{pmatrix} = 24 + 12 - 54 = -18$$

$$\therefore \text{acute angle} = \cos^{-1} \left| \frac{-18}{19 \times 13} \right| = \cos^{-1} \frac{18}{247} = 85.8^\circ$$

$$\mathbf{c} \quad \left| \begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix} \right| = \sqrt{1+1+9} = \sqrt{11}, \quad \left| \begin{pmatrix} 2 \\ -5 \\ 3 \end{pmatrix} \right| = \sqrt{4+25+9} = \sqrt{38}, \quad \begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ -5 \\ 3 \end{pmatrix} = 2 + 5 + 9 = 16$$

$$\therefore \text{acute angle} = \cos^{-1} \left| \frac{16}{\sqrt{11} \times \sqrt{38}} \right| = \cos^{-1} \frac{16}{\sqrt{11} \sqrt{38}} = 38.5^\circ$$

$$\mathbf{d} \quad \left| \begin{pmatrix} -4 \\ -6 \\ 7 \end{pmatrix} \right| = \sqrt{16+36+49} = \sqrt{101}, \quad \left| \begin{pmatrix} 5 \\ -1 \\ -8 \end{pmatrix} \right| = \sqrt{25+1+64} = \sqrt{90}, \quad \begin{pmatrix} -4 \\ -6 \\ 7 \end{pmatrix} \cdot \begin{pmatrix} 5 \\ -1 \\ -8 \end{pmatrix} = -20 + 6 - 56 = -70$$

$$\therefore \text{acute angle} = \cos^{-1} \left| \frac{-70}{\sqrt{101} \times \sqrt{90}} \right| = \cos^{-1} \frac{70}{\sqrt{101} \sqrt{90}} = 42.8^\circ$$

17 a  $\overrightarrow{AB} = (6\mathbf{i} + 5\mathbf{j} + \mathbf{k}) - (5\mathbf{i} + 8\mathbf{j} - \mathbf{k}) = \mathbf{i} - 3\mathbf{j} + 2\mathbf{k}$

$\therefore \mathbf{r} = 5\mathbf{i} + 8\mathbf{k} - \mathbf{k} + \lambda(\mathbf{i} - 3\mathbf{j} + 2\mathbf{k})$

b  $5 + \lambda = 4 - 5\mu$  (1)

$8 - 3\lambda = -3 + \mu$  (2)

$-1 + 2\lambda = 5 - 2\mu$  (3)

$3 \times (1) + (2) \Rightarrow 23 = 9 - 14\mu$

$\mu = -1, \lambda = 4$

check (3)  $-1 + 2(4) = 5 - 2(-1)$

true  $\therefore$  intersect

pos. vector of int. =  $9\mathbf{i} - 4\mathbf{j} + 7\mathbf{k}$

c  $|(\mathbf{i} - 3\mathbf{j} + 2\mathbf{k})| = \sqrt{1+9+4} = \sqrt{14}$

$|(-5\mathbf{i} + \mathbf{j} - 2\mathbf{k})| = \sqrt{25+1+4} = \sqrt{30}$

$(\mathbf{i} - 3\mathbf{j} + 2\mathbf{k}) \cdot (-5\mathbf{i} + \mathbf{j} - 2\mathbf{k}) = -5 - 3 - 4 = -12$

acute angle =  $\cos^{-1} \left| \frac{-12}{\sqrt{14} \times \sqrt{30}} \right| = \cos^{-1} \frac{6}{\sqrt{105}} = 54.2^\circ$  (1dp)

18  $\lambda = \frac{x-2}{3} = \frac{y}{2} = \frac{z+5}{-6}$

$\mu = \frac{x-4}{-4} = \frac{y+1}{7} = \frac{z-3}{-4}$

$x = 2 + 3\lambda, y = 2\lambda, z = -5 - 6\lambda$

$x = 4 - 4\mu, y = -1 + 7\mu, z = 3 - 4\mu$

$\mathbf{r} = 2\mathbf{i} - 5\mathbf{k} + \lambda(3\mathbf{i} + 2\mathbf{j} - 6\mathbf{k})$

$\mathbf{r} = 4\mathbf{i} - \mathbf{j} + 3\mathbf{k} + \mu(-4\mathbf{i} + 7\mathbf{j} - 4\mathbf{k})$

$|3\mathbf{i} + 2\mathbf{j} - 6\mathbf{k}| = \sqrt{9+4+36} = 7$

$|(-4\mathbf{i} + 7\mathbf{j} - 4\mathbf{k})| = \sqrt{16+49+16} = 9$

$(3\mathbf{i} + 2\mathbf{j} - 6\mathbf{k}) \cdot (-4\mathbf{i} + 7\mathbf{j} - 4\mathbf{k}) = -12 + 14 + 24 = 26$

$\therefore$  acute angle =  $\cos^{-1} \left| \frac{26}{7 \times 9} \right| = \cos^{-1} \frac{26}{63} = 65.6^\circ$

19 a  $7 + 2\lambda = -4 + 5\mu$  (1)

$-\lambda = 7 - 4\mu$  (2)

$-2 + 2\lambda = -6 - 2\mu$  (3)

$(1) - (3) \Rightarrow 9 = 2 + 7\mu$

$\mu = 1$

$\therefore A(1, 3, -8)$

b  $|2\mathbf{i} - \mathbf{j} + 2\mathbf{k}| = \sqrt{4+1+4} = 3$

$|5\mathbf{i} - 4\mathbf{j} - 2\mathbf{k}| = \sqrt{25+16+4} = 3\sqrt{5}$

$(2\mathbf{i} - \mathbf{j} + 2\mathbf{k}) \cdot (5\mathbf{i} - 4\mathbf{j} - 2\mathbf{k}) = 10 + 4 - 4 = 10$

acute angle =  $\cos^{-1} \left| \frac{10}{3 \times 3\sqrt{5}} \right| = \cos^{-1} \left( \frac{2}{9}\sqrt{5} \right) = 60.2^\circ$  (1dp)

c  $7 + 2\lambda = 5 \Rightarrow \lambda = -1$

sub.  $\lambda = -1$  in eqn for  $l$

$\mathbf{r} = 7\mathbf{i} - 2\mathbf{k} - (2\mathbf{i} - \mathbf{j} + 2\mathbf{k}) = 5\mathbf{i} + \mathbf{j} - 4\mathbf{k}$

$\therefore B$  lies on  $l$

d  $\overrightarrow{AB} = (5\mathbf{i} + \mathbf{j} - 4\mathbf{k}) - (\mathbf{i} + 3\mathbf{j} - 8\mathbf{k}) = 4\mathbf{i} - 2\mathbf{j} + 4\mathbf{k}$

$|\overrightarrow{AB}| = \sqrt{16+4+16} = 6$

$\therefore$  dist. of  $B$  from  $m = 6 \sin 60.20^\circ = 5.21$  (3sf)

$$20 \quad \mathbf{a} \quad \overrightarrow{AB} = (11\mathbf{i} + 5\mathbf{j} + \mathbf{k}) - (9\mathbf{i} + 6\mathbf{j}) = 2\mathbf{i} - \mathbf{j} + \mathbf{k}$$

$$\overrightarrow{OC} = 9\mathbf{i} + 6\mathbf{j} + \lambda(2\mathbf{i} - \mathbf{j} + \mathbf{k})$$

$$= \overrightarrow{OA} + \lambda\overrightarrow{AB}$$

$\therefore C$  lies on  $l$

$$\mathbf{b} \quad [(9 + 2\lambda)\mathbf{i} + (6 - \lambda)\mathbf{j} + \lambda\mathbf{k}] \cdot [2\mathbf{i} - \mathbf{j} + \mathbf{k}] = 0$$

$$2(9 + 2\lambda) - (6 - \lambda) + \lambda = 0$$

$$\lambda = -2$$

$$\mathbf{c} \quad \text{sub. } \lambda = -2 \text{ in } \overrightarrow{OC}$$

$$\therefore 5\mathbf{i} + 8\mathbf{j} - 2\mathbf{k}$$

$$21 \quad \mathbf{a} \quad [(-4 + \lambda)\mathbf{i} + (2 + 3\lambda)\mathbf{j} + (7 - 4\lambda)\mathbf{k}] \cdot [\mathbf{i} + 3\mathbf{j} - 4\mathbf{k}] = 0$$

$$(-4 + \lambda) + 3(2 + 3\lambda) - 4(7 - 4\lambda) = 0$$

$$\lambda = 1$$

$$\therefore (-3, 5, 3)$$

$$\mathbf{b} \quad [(7 + 6\lambda)\mathbf{i} + (11 - 9\lambda)\mathbf{j} + (-9 + 3\lambda)\mathbf{k}] \cdot [6\mathbf{i} - 9\mathbf{j} + 3\mathbf{k}] = 0$$

$$6(7 + 6\lambda) - 9(11 - 9\lambda) + 3(-9 + 3\lambda) = 0$$

$$\lambda = \frac{2}{3}$$

$$\therefore (11, 5, -7)$$